

# AN ANALYSIS OF $V_{50}$ BALLISTIC LIMIT RESULTS ADJUSTING 1ST SHOT VELOCITY, STEP-UP STEP-DOWN INCREMENTS, TRUTH CHARACTERISTICS AND VELOCITY CONTROL DISTRIBUTIONS

C. Andres<sup>1</sup>, W. Boughers<sup>1</sup>

<sup>1</sup>RAM/ILS Engineering and Analysis Division, U.S. Army Aberdeen Test Center (ATC), 400 Collieran Road, Aberdeen Proving Ground, Maryland 21005, USA, [craig.d.andres.civ@mail.mil](mailto:craig.d.andres.civ@mail.mil)

**Abstract.** Perforation performance of body armor is typically evaluated by  $V_{50}$  ballistic limit testing. Shot velocities are sequentially controlled by a up/down method to populate a sample containing results on both sides of the true value of the  $V_{50}$  velocity. The  $V_{50}$  velocity is the velocity at which the probability of perforation is 50 percent. Test protocols control starting velocities and the up/down method, but acquisition of the desired shot velocity is limited by the velocity control of the firing system. Since allowable sample sizes can be relatively small, it is questionable in such cases whether perforation and non-perforation results are sufficiently balanced about the true  $V_{50}$  response of the test item to yield accurate results. Bias in the result likely exists depending on how far away from and on which side of the true  $V_{50}$  the first shot takes place. Further, insufficient data may exist to yield an accurate  $V_{50}$  result if the zone of mixed results for the test item is relatively large. The analysis reveals how the  $V_{50}$  estimate is influenced by varying the discussed parameters relative to a realistic soft armor binary response relationship defined by a logistic regression function. The nominal parameter settings and function are used as the true underlying response curve for the simulations conducted. Different methods of calculating  $V_{50}$  are also examined which include simple averaging of six round and ten round samples as well as a logistic regression solution. Larger samples are examined as well as the influence of step size on the  $V_{50}$  result. The truth set is varied from a step response to a characteristic logistic distribution in order to analyze the effects of the zone of mixed results. Trends are presented relative to the  $V_{50}$  truth set and are used to take a broad look at the test methodology as a whole. Recommendations are made to improve the consistency and accuracy of the  $V_{50}$  test methodology.

## 1. INTRODUCTION

Ballistic testing is a necessary part of the characterization and monitoring of performance of most body armor on the market. The velocity at which complete penetrations occur 50 percent of the time, called  $V_{50}$ , is one of the common metrics used to evaluate performance. For the  $V_{50}$  test method analyzed in this paper, shots are fired using a standard up/down procedure to determine subsequent target velocities. After the initial shot, these targeted velocities are stepped up or down by a fixed increment based on the penetration result of the prior shot. Once a specific set of criteria are met, the test is ended and an estimate of  $V_{50}$  is calculated. The typical case uses the three lowest velocities with complete penetrations and the three highest velocities with partial penetrations within a 38 mps (125 fps) window. The  $V_{50}$  estimate is calculated by taking the simple mean of the measured velocity for these six points. Less typically, five results from each penetration outcome are used if within a 46 mps (150 fps) window. The goal of this test method is to provide an estimate that establishes that a product meets the performance requirement while minimizing the number of shots needed for each test to keep the cost of the test low.

The test method has some parameters that are controlled and others that are not. Testers strive to control the shot velocity, but there is variation between the velocity targeted and the velocity achieved. This promotes variation in the actual step size. Testers do determine the starting velocity, which has been typically chosen following MIL-STD-662F (ref.1) as either an estimated  $V_{50}$  or a fixed value at 30 mps (100 fps) above the  $V_{50}$  requirement. One of the goals of this paper is to gain a better understanding of

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how velocity control and choice of starting velocity can influence both the  $V_{50}$  estimate and the average number of shots needed to complete each test. Though parameters affecting  $V_{50}$  estimates can be intuitively identified, the quality and sensitivity of the result relative to these contributing parameters are seldom quantified. This work will examine error associated with the method and sensitivities of the result to contributing parameters using parametric modeling.

## **2. MODEL**

### **2.1 Approach**

The statistics software package, R, was used to encode the test method, define the true  $V_{50}$  distribution for the product, and define the velocity control distribution, all described below. Simulations were performed varying velocity control, step increment size, truth characteristics and starting velocity. Underlying assumptions for the simulations include:  $V_{50}$  is constant across the product being tested, the probability of a complete penetration follows a logistic curve, the difference between the targeted velocity and actual striking velocity follows a normal distribution with mean zero and the standard deviation is typically 6 mps (20 fps). For some simulations, a step function for the response was approximated by adjusting the logistic curve parameters, and perfect velocity control was achieved by setting the standard deviation for the normal distribution at zero. Some of these assumptions are not born out in real test data but are used to provide comparisons against ideal settings. In most cases, 2000 simulations were used to give enough character to plots without over populating points on the graphs. For trends analysis, 1000 simulations were performed at each step increment.

### **2.2 Current test method**

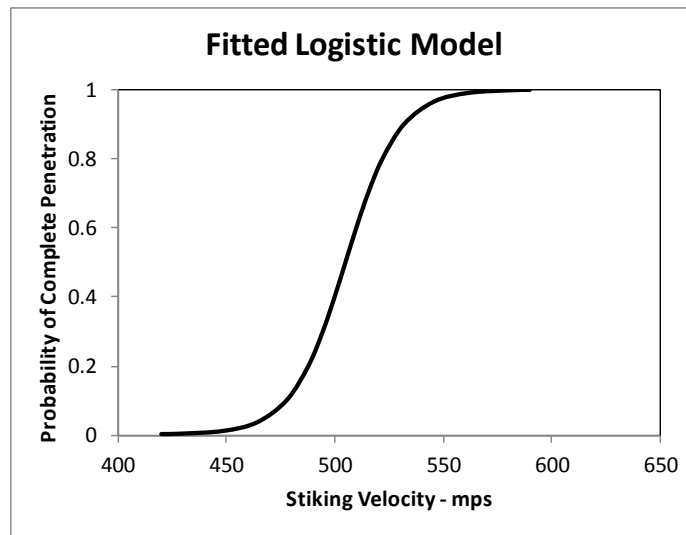
A common procedure used to acquire a  $V_{50}$  estimate, found in MIL-STD-662F (see ref. 1), makes use of a minimum required  $V_{50}$  limit as reference point.  $V_{50}$  estimates below this limit are considered failing or uncharacteristic. The first shot velocity targeted in a six shot ballistic limit test is typically a set increment above this minimum value. If a complete penetration is not achieved on the first shot, the velocity is stepped up by an increment of 15 mps (50 fps). This is repeated until a complete penetration is achieved, at which point the next targeted velocity will step down 15 mps (50 fps). This is termed a reversal because of the switch in the penetration result and the direction in which the next velocity will be targeted. Targeted velocities for each shot are stepped up or down based on the penetration binary response. A partial penetration (P) increases the targeted velocity of the next shot, and a complete penetration (C) decreases it. MIL-STD-662F makes one exception to this simple logic which only applies to the first shot. In this case, if a complete penetration occurs, the step down increment is 15 mps (50 fps) to 30 mps (100 fps). The model in this analysis uses 30 mps (100 fps). This simple step up/down process is continued until the test requirements are achieved. As mentioned in section 1, the test is concluded once the three highest partial and three lowest complete penetrations are achieved within a 38 mps (125 fps) window or spread. However, in the model once ten or more shots are present, if five partials and five completes can be acquired within a 46 mps (150 fps) spread, these results are averaged to find the  $V_{50}$  estimate. As a last resort, if neither window is achieved by sixteen shots, this study has added a logistics regression result using all 16 shots (assumes a large soft armor test item) to estimate the  $V_{50}$ . This entire procedure as described was encoded as the baseline test methodology for the simulations under discussion.

### **2.3 True response function characterization**

A response curve was derived from  $V_{50}$  and  $V_{02}$  (2% probability of complete penetration) information in reference 2 (Table 2.2) for a particular body armor design (soft armor) and is shown in Figure 1. A logistic regression function was used where the probability of complete penetration as a function of velocity is

$\pi(V) = \exp(\alpha + \beta V) / (1 + \exp(\alpha + \beta V))$ . When  $\beta > 0$  the logistic regression curve has the shape of a cumulative distribution function (cdf) for a logistic distribution where  $\mu = -\alpha/\beta$  and  $\sigma = \pi/(|\beta|\sqrt{3})$ . The logistic regression fit to the data produces coefficients  $\alpha = -41.07$  and  $\beta = 0.02479$ , which convert to  $\mu = 505$  mps (1657 fps) and  $\sigma = 22.3$  mps (73.2 fps). The estimate of  $\mu$  from this fitted model is taken to be the true value of the  $V_{50}$  used for the simulations. Subsequent simulations were conducted using this regression model to define the true underlying distribution for the response (which will be heretofore referred to as the “truth” or “baseline response”) which is used to represent the penetration performance of the soft armor item for the simulations.

The response shown in Figure 1 does not take into account the effects of the prior shot history on any given item. Such shot dependency is not accounted for in the simulations and is beyond the scope of this analysis. Nonetheless, the simulations from this study serve to illustrate the potential effects of changing test parameters on  $V_{50}$  results.



**Figure 1.** Probability plot of the baseline response

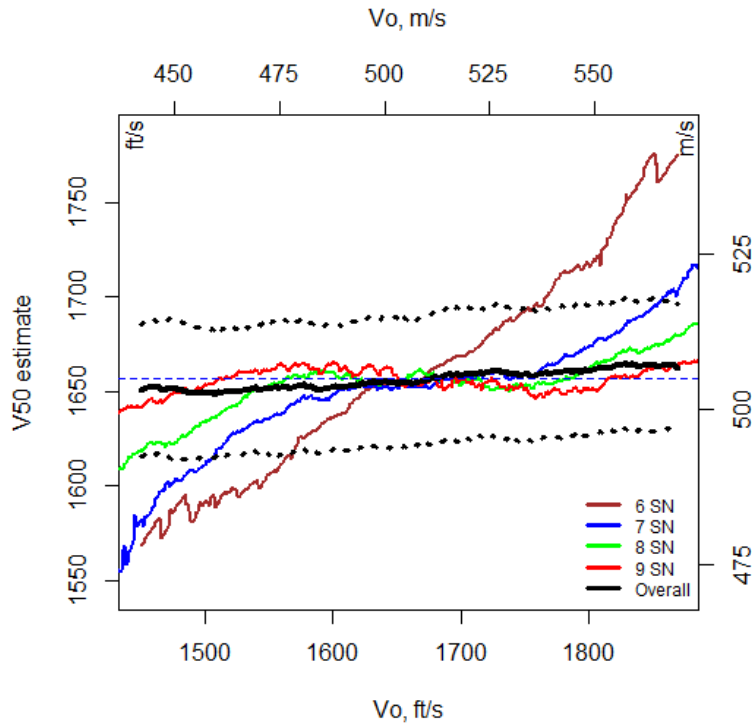
## 2.4 Velocity control and penetration assignment

The velocity control (delta from targeted velocity) of the threat associated with the response curve shown in section 2.3 was assumed to be normally distributed, with mean zero and the standard deviation values were varied using 0 mps, 6 mps (20 fps), and 12 mps (40fps). The 6 mps value was taken to be the baseline (expected) value. For a given shot simulation, a velocity,  $V_t$ , was targeted and a calculation of the actual shot velocity,  $V_a$ , was made where  $\Delta V = V_a - V_t$  which is defined for the simulation as a normally distributed random variable defined by the standard deviation (also referred to as velocity control). A penetration result was assigned to the velocity  $V_a$  based on the plot shown in Figure 1 using a random function weighted by the local probability. The step increment was applied to  $V_a$  to formulate the next value of  $V_t$  and the process repeated again until a shot criterion was met.

## 3. RESULTS

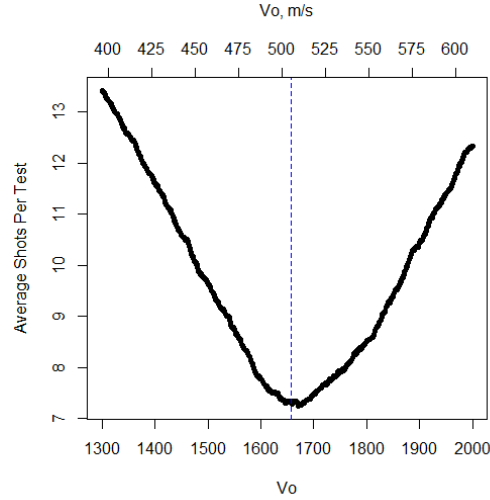
### 3.1 Effects of starting velocity on $V_{50}$ result

The starting velocity was varied from 442 mps (1450 fps) to 579 mps (1900 fps) using the baseline settings described in section 2. The number of simulations was set at 1000 for each 0.3 mps (1 fps) increment in this range. The results of the effect on the  $V_{50}$  estimate are shown in Figure 2 below. The lines in the figure represent the mean of the  $V_{50}$  estimates at each incremented starting velocity  $V_0$ . The blue dashed horizontal line represents the true  $V_{50}$  for the simulation which was defined earlier as  $\mu_t = 505$  mps (1657 fps). The velocity standard deviation used for velocity control was set at the baseline value of 6 mps (20 fps). The solid black line is the mean for all 1000 simulated results at each velocity (all inclusive mean, 6 to 16 shot solutions, individual results for 10 to 16 shot solutions not plotted to reduce clutter), with the black dotted lines being that mean plus and minus one standard deviation ( $\mu \pm \sigma$ ). The brown, blue, green, and red lines represent the mean  $V_{50}$  calculated for the case where exactly 6, 7, 8, and 9 (respectively) shots were needed to satisfy the requirements to get a valid estimate for  $V_{50}$  (3 partials, 3 completes within 38 mps (125 fps)). In these cases, and when  $V_0$  is relatively low, the resulting  $V_{50}$  is negatively biased. The opposite holds true when  $V_0$  is high. The interesting finding is that the low shot number solutions have a very noticeable bias. For example, when  $V_0 = 457$  mps (1500 fps), the average bias of all test results is approximately 1.5 mps (5 fps) on the low side; however, the average of the estimates achieved in the first six shots can be substantially biased on the low side as much as 25 mps (82 fps). The distribution of shot number solutions at different settings will be shown in the next section. A case at  $V_0=457$  mps and velocity control at  $SD = 6$  mps as in the example just given will be shown.



**Figure 2.** Influence of starting velocity on  $V_{50}$  estimate

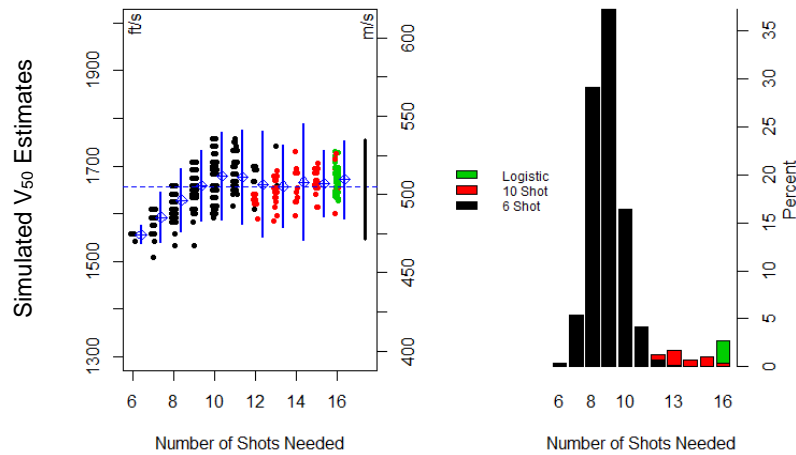
In Figure 3, the average number of shots per test (SPT) is shown as a function of starting velocity. It is seen that the efficiency of the test decays as the starting velocity diverges from the truth value (blue dotted line through 505 mps). For example, using starting velocity  $V_0 = 457$  mps (1500 fps), one can expect to need approximately 2.3 shots more per test on average (under the assumptions of the simulation) than when  $V_0 = \text{true } V_{50}$ .



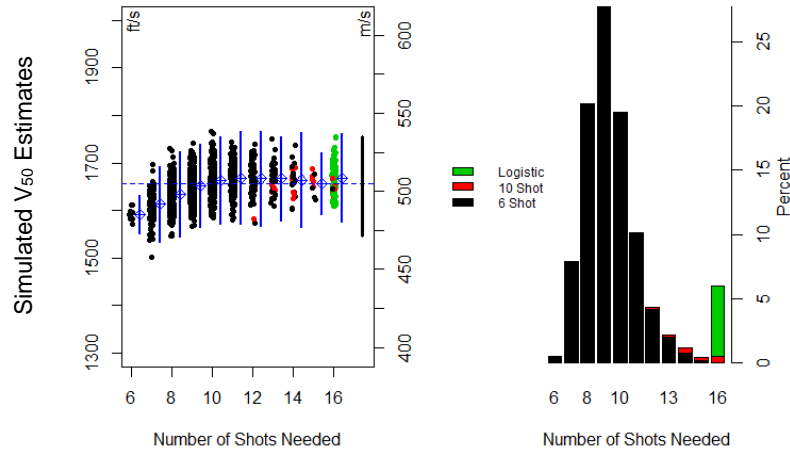
**Figure 3.** Average number of shots per test, as a function of starting velocity, baseline settings

### 3.2 Effect of velocity control on $V_{50}$ result

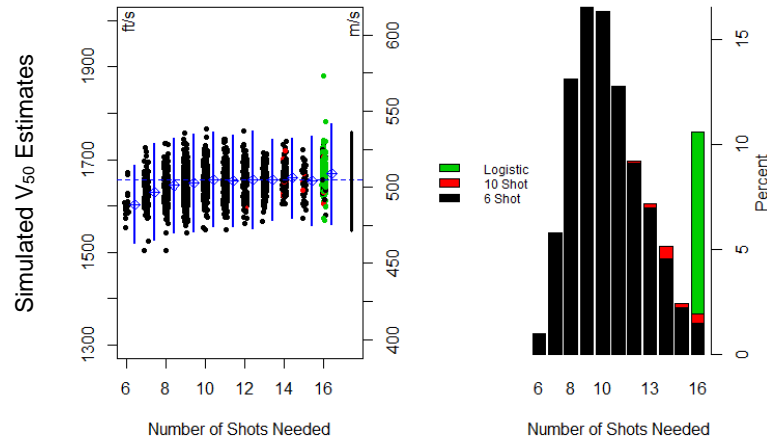
The effect of velocity control on  $V_{50}$  estimates is shown in Figures 4 to 6 below. Three different velocity control standard deviation values were examined, which were 0 mps, 6 mps (20 fps), and 12 mps (40 fps). The results are from 2000 simulations at each of these values. Black dots are a 3P-3C (6 shot sample) solution, red dots are a 5P-5C (10 shot sample) solution (used when number of shots  $\geq 10$  and spread criterion met, otherwise 3P-3C solution is used if its spread criterion is met) and green dots are a logistic regression solution (used when the other two solution types do not converge by shot 16). The blue symbols and solid lines show the mean and 3 sigma variation in the  $V_{50}$  estimate for each number of shots and the black lines are the 3 sigma variation for overall results (6 to 16 shot solutions). It is seen that good velocity control (lower standard deviation between targeted and actual velocities) promotes a bias in the  $V_{50}$  estimates for low shot number solutions. As velocity control degrades there is less bias in the  $V_{50}$  estimate but increasing variation, especially in low shot number solutions. The starting velocity used in Figures 4 to 6 was 457 mps (1500 fps). Less or no bias occurs when using starting values near the true  $V_{50}$  of 505 mps (1657 fps) which can be seen when Figure 5 is compared with Figure 10 from the next section.



**Figure 4.**  $V_{50}$  estimate with perfect velocity control  $SD = 0$ , baseline logistic response curve, starting velocity  $V_0 = 457$  mps (1500 fps)



**Figure 5.** V<sub>50</sub> estimate when velocity control SD = 6 mps (20 fps), baseline logistic response curve, starting velocity V<sub>0</sub> = 457 mps (1500 fps)



**Figure 6.** V<sub>50</sub> estimate when velocity control SD = 12 mps (40 fps), baseline logistic response, starting velocity V<sub>0</sub> = 457 mps (1500 fps)

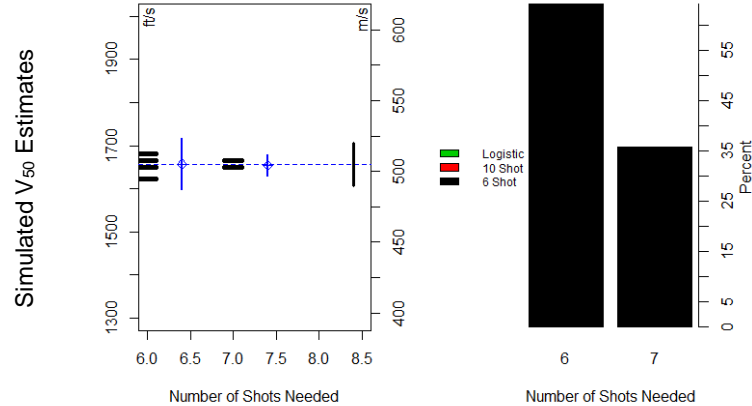
The average number of shots per test (SPT) increases as velocity control degrades (SPT = 9.2, 9.7, and 10.9, Figures 4-6 respectively). When velocity control SD = 12 mps (40 fps) it is interesting to note that the number of 16 shot solutions increases dramatically, no doubt because it is becoming harder to achieve velocities in the narrow spread windows required..

### 3.3 Effects of varying the response parameters from the truth

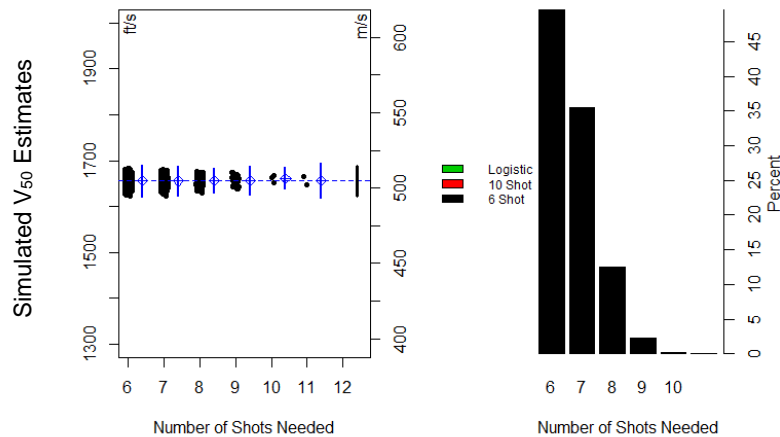
In Figures 7 to 10 below, a progression from deterministic velocity control and truth response characteristics to the more realistic baseline characteristics used are examined. The parameters in the logistic function were adjusted to make it nearly a step function to get a deterministic response while maintaining the true V<sub>50</sub> = 505 mps. The results in each figure were from 2000 simulations and the starting velocities were fixed at V<sub>0</sub> = 505 mps (1657 fps) which is the true V<sub>50</sub> value. It is seen in Figure 7 that in absence of all variation (response curve and velocity control), convergence for a 3P-3C result occurs in 7 shots or less every time (average SPT = 6.4). When variation due to velocity control is added, see Figure 8, the average SPT increases to 6.7. In Figure 9, the velocity control is made perfect again but the baseline

response curve from Figure 1 is used. The variation in the estimate is greatly increased as a result and the average SPT=6.8. The baseline velocity SD of 6 mps (20 fps) is added back in as shown in Figure 10 and the average SPT reaches 7.4.

In Figure 11, the starting velocity was lowered to 457 mps (1500 fps), a step function response was used and the velocity control was set at SD = 6 mps (20 fps). It is seen that the shot distribution shifts right (less efficient) as the starting velocity is lowered when compared to Figure 8. Comparing Figure 11(step response) to Figure 5 (baseline response), it is seen that much of the added bias in the  $V_{50}$  estimates at the lower starting velocity, and the added variation in  $V_{50}$  estimates at all levels, occur in relation to the increased variation in the underlying response curve. Thus it could be said, for low shot number solutions, that the test method has difficulties producing adequate data for accurate  $V_{50}$  estimates when the variation in the truth set is relatively large (large zone of mixed results relative to deterministic case) and the starting velocity is approximately 50 mps away from the true  $V_{50}$  value. Further, when the starting velocity is at the true  $V_{50}$ , the variation in the baseline truth set used contributed to an increase in the variation in the  $V_{50}$  estimate (see Figure 10 vertical blue lines) by a factor of about 3 when compared to the use of a deterministic truth set (see Figure 8 vertical blue lines).

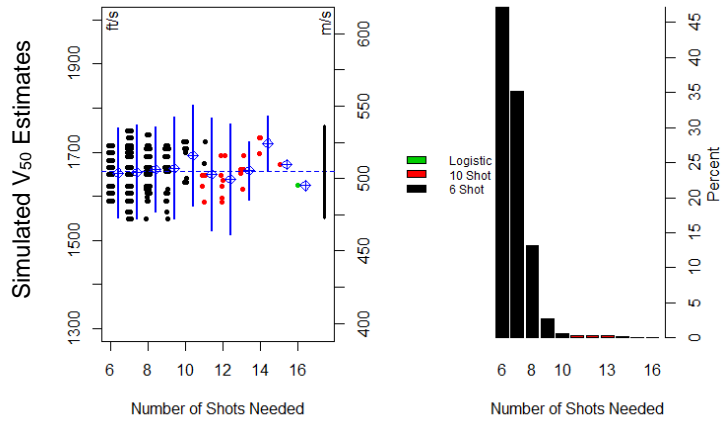


**Figure 7.**  $V_{50}$  estimate, perfect velocity control SD = 0, step function response, starting velocity  $V_0$  = 505 mps (1657 fps), SPT result = 6.4

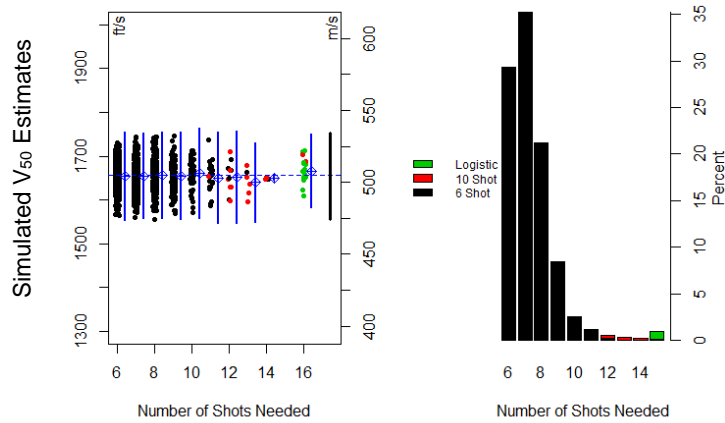


**Figure 8.**  $V_{50}$  estimate, velocity control SD = 6 mps (20 fps), step function response, starting velocity  $V_0$  = 505 mps (1657 fps), SPT result = 6.7

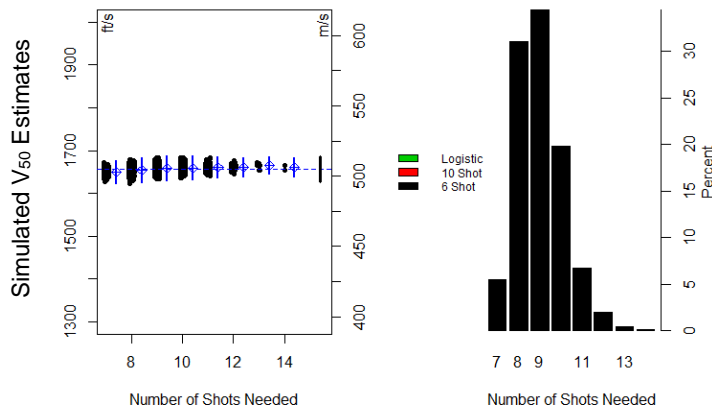




**Figure 9.**  $V_{50}$  estimate, perfect velocity control  $SD = 0$ , baseline logistic response curve, starting velocity  $V_0 = 505$  mps (1657 fps), SPT result = 6.8



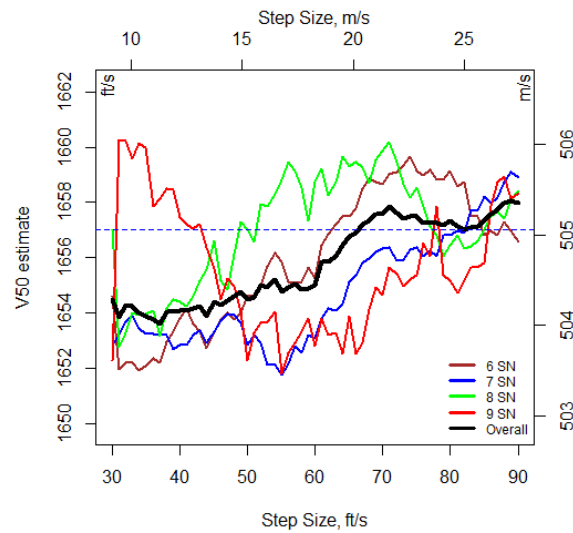
**Figure 10.**  $V_{50}$  estimate, velocity control  $SD = 6$  mps (20 fps), baseline logistic response curve, starting velocity  $V_0 = 505$  mps (1657 fps), SPT result = 7.4



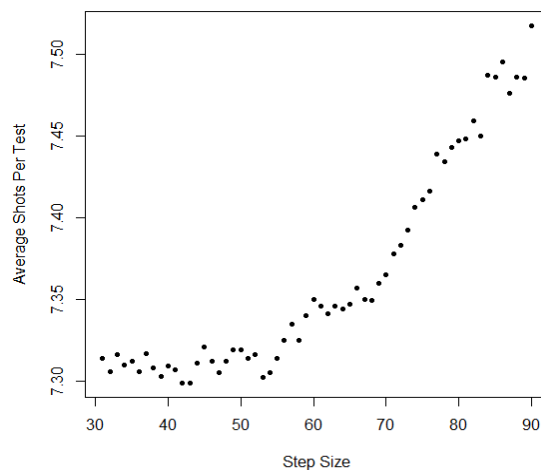
**Figure 11.**  $V_{50}$  estimate, velocity control  $SD = 6$  mps (20 fps), step function response, starting velocity  $V_0 = 457$  mps (1500 fps), SPT result = 9.0

### 3.4 Effect of step size on $V_{50}$ result

The effect of step size was examined when the starting velocity equals the  $V_{50}$  truth value. The results are shown in Figure 12. The brown, blue, green, and red lines represent the mean  $V_{50}$  calculated for the case where exactly 6, 7, 8, and 9 shots (respectively) were needed to satisfy the requirements to get a valid estimate for  $V_{50}$  (3 partials, 3 completes within 38 mps (125 fps)). Some bias (small, < 1mps below 505 mps truth value) is seen in the  $V_{50}$  result for the baseline step size of 15 mps (50 fps). This is likely due to the 30 mps (100 fps) step down which may be used if a C occurs on the first shot. Larger step sizes tend to eliminate the bias while increasing the variation in the  $V_{50}$  estimate (from results not shown). Figure 13 shows that step size can be adjusted to optimize test efficiency as realized by the average SPT trend shown.



**Figure 12.** Effect of step size on the  $V_{50}$  estimate, velocity control SD = 6 mps (20 fps), baseline logistic response curve, starting velocity  $V_0$  = 505 mps (1657 fps)



**Figure 13.** Effect of step size on average number of shots per test using settings from Figure 12

#### 4. CONCLUSIONS

A parametric model was constructed to simulate the step up/down approach commonly used in testing to achieve a  $V_{50}$  estimate result for a body armor test item. When the starting velocity for the shot sequence diverged from the  $V_{50}$  truth value, it was demonstrated that a bias in the resulting  $V_{50}$  estimate could be introduced. A low-side starting velocity yielded a low-side bias in the estimate and a high-side starting velocity a high-side bias. This bias is most pronounced in lowest shot number solutions. It was also found that the shots needed per test (SPT) increases as the starting velocity moves away from the true  $V_{50}$  value hence decreasing test efficiency which would increase test cost. As velocity control improves test efficiency improves (less shots required), but in some cases bias introduced by test methodology (low starting velocity) can also increase as a result of improved velocity control. It was also found that the higher the variation in the response truth set (or a larger zone of mixed results), the higher the variation in the  $V_{50}$  estimate, less efficient the test, and the more prone the test result is to test method induced bias in the  $V_{50}$  estimate. It was determined that selection of an optimum step size can help improve test efficiency. Use of the model shows there is potential to optimize test parameters to make gains in the efficiency of testing, hence cost, as well as to make gains in the accuracy and precision of the  $V_{50}$  estimate.

In practice the findings show that testers/evaluators need to be aware of the potential implications of using starting velocities that deviate from the true  $V_{50}$  value. The model suggests that the consistent use of starting velocities that approximate true  $V_{50}$  performance (historical or short run) will lead to the best test accuracy, test efficiency and reproducibility in the result.

The conclusions of this paper represent the personal opinion of the researchers and do not imply an official position from ATC, the U.S. Army or U.S. Department of Defense.

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